Under the external forces $q_x = 1{,}5\tau$, $q_y = 2\tau$ the PD boundary obtained from the Galin solution encloses the circular hole. It is shown by dashes in Fig.1. Values of p_L corresponding to the solution mentioned are given by dashes in Fig.2.

It is seen that the Galin solution of the problem and the method of successive approximations are practically in agreement for load values $q_x = 1{,}5\tau$, $q_y = 2\tau$. This means that when the conditions of monotonicity of the PD development and the constraint on load asymmetry are satisfied

$$q_y - q_x \leqslant 0{,}82\tau \tag{3.1}$$

the solution obtained by Galin also corresponds to those loading trajectories for which incomplete enclosure of the circular hole by the PD is possible during strain. Under the conditions mentioned, the Galin solution is obviously independent of the history of the change in external forces.

We consider the loading program $(q_x/\tau;\ q_y/\tau) = (1{,}2;\ 2),\ (1{,}9;\ 2{,}6),\ (2;\ 3)$, terminated by forces for which the condition of PD enclosure of the circular hole is satisfied but the condition imposing the constraint (3.1) on the load asymmetry is spoiled. The PD and the values of the WD density are shown in Figs.3 and 4 for the load $q_x = 1{,}2\tau, q_y = 2\tau$ (curve *1*); $q_x = 1{,}9\tau, q_y = 2{,}6\tau$ (curve *2*), $q_x = 2\tau, q_y = 3\tau$ (curve *3*).

The PD boundary and the graph of WD density values obtained on the basis of the Galin solution for $q_x = 2\tau, q_y = 3\tau$ are shown by dashes.

It follows from the example presented that if the load asymmetry does not safisfy condition (3.1), then the state of stress and strain of a plane with a circular hole depends on the loading trajectory (the history of the change in external forces).

The author is grateful to M.Ya. Leonov for his interest.

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Translated by M.D.F.

PMM U.S.S.R.,Vol.51,No.5,pp.670-674,1987
Printed in Great Britain

0021-8928/87 $10.00+0.00

ELASTIC EQUILIBRIUM OF CIRCULAR PIECEWISE-HOMOGENEOUS MEDIA WITH A DIAMETRAL CRACK*

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A method is proposed for solving boundary value problems of elasticity theory for circular piecewise-homogeneous media with a symmetric diametral crack, based on the application of vector relationships between the basis solutions of the equilibrium equations in polar and elliptic coordinates. The realization of this method results in infinite systems of linear algebraic equations of the second kind with exponentially decreasing matrix coefficients.

The problem of the equilibrium of a two-component piecewise-homogeneous plane with a symmetric diametral crack is considered in an inner homogeneous domain. Asymptotic formulas for the stress intensity coefficients are obtained by expansion in a small parameter.

1. The elliptic coordinates are related to the Cartesian coordinates by the formulas

**Prikl.Matem.Mekhan.*,51,5,853-857,1987

$$x = a \operatorname{ch} \xi \cos \theta, \quad y = a \operatorname{sh} \xi \sin \theta \quad (0 \leqslant \xi < \infty, \quad 0 \leqslant \theta < 2\pi, \quad a > 0)$$

The equation $\xi = \text{const}$ determines the family of ellipses with foci at the points $y = 0$, $x = \pm a$. In the limit case $\xi = 0$ the ellipse degenerate into the focal segment $y = 0$, $|x| \leqslant a$.

The Lamé vector equilibrium equation for a homogeneous isotropic body has the form

$$\frac{1}{1-2\nu} \operatorname{grad} \operatorname{div} \mathbf{u} + \Delta \mathbf{u} = 0 \tag{1.1}$$

Here $\mathbf{u}$ is the elastic displacements vector and ν is Poisson's ratio. The bulk forces are omitted in (1.1) (they can be taken into account without any special difficulties in the presence of the general solution of the homogeneous Lamé equation). All the formulas presented here correspond to the plane strain case. In the case of a generalized plane state of stress, Poisson's ratio ν should be replaced by the quantity $\nu/(1+\nu)$.

The basis periodic solutions of (1.1) in elliptic and polar coordinates can be represented in the form of the complex-valued vector functions

$$\mathbf{u}_{1,n}^{(e)} = \operatorname{ch} n (\xi + i\theta)(\mathbf{e}_x + i\mathbf{e}_y) \tag{1.2}$$

$$\mathbf{u}_{2,n}^{(e)} = (y \operatorname{grad} - \varkappa \mathbf{e}_y) \operatorname{ch} n (\xi + i\theta)$$

$$\mathbf{u}_{3,n}^{(e)} = e^{-n\xi} e^{in\theta} (\mathbf{e}_x - i\mathbf{e}_y) \tag{1.3}$$

$$\mathbf{u}_{4,n}^{(e)} = (y \operatorname{grad} - \varkappa \mathbf{e}_y) e^{-n\xi} e^{in\theta} \quad (n = 0, 1, 2, \ldots)$$

$$\mathbf{u}_{1,k}^{(p)} = \rho^k e^{i(k+1)} \varphi (\mathbf{e}_\rho + i\mathbf{e}_\varphi) \tag{1.4}$$

$$\mathbf{u}_{2,k}^{(p)} = (y \operatorname{grad} - \varkappa \mathbf{e}_y)(2\rho^k e^{ik\varphi}) + i(k-\varkappa)\mathbf{u}_{1,k}^{(p)} = \\ = [i(k-\varkappa)\mathbf{e}_\rho - (k+\varkappa)\mathbf{e}_\varphi]\rho^k e^{i(k-1)\varphi}$$

$$\mathbf{u}_{3,k}^{(p)} = \rho^{-k} e^{i(k-1)\varphi} (\mathbf{e}_\rho - i\mathbf{e}_\varphi)$$

$$\mathbf{u}_{4,k}^{(p)} = (y \operatorname{grad} - \varkappa \mathbf{e}_y)(2\rho^{-k} e^{ik\varphi}) + i(k+\varkappa)\mathbf{u}_{3,k}^{(p)} = \\ = [i(k+\varkappa)\mathbf{e}_\rho + (k-\varkappa)\mathbf{e}_\varphi]\rho^{-k} e^{i(k+1)\varphi} \quad (k = 0, 1, 2, \ldots)$$

$$\mathbf{u}_0^{(p)} = D_0 \left[\varkappa \ln \rho (\mathbf{e}_\rho + i\mathbf{e}_\varphi) - \frac{1}{2}(\mathbf{e}_\rho - i\mathbf{e}_\varphi)\right] e^{i\varphi} \tag{1.5}$$

($\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_\rho$, $\mathbf{e}_\varphi$ are the directions of the Cartesian and polar coordinate systems $x = \rho \cos \varphi$, $y = \rho \sin \varphi$, $\varkappa = 3 - 4\nu$, $D_0 = \text{const}$).

The solutions (1.2) and (1.3), which are regular, respectively, in any finite domain and in a plane with the slit $\xi = 0$, are connected with the solutions (1.4) by the relationships ($\binom{m}{p}$ are binomial coefficients)

$$\mathbf{u}_{1,2n-1}^{(p)} = 2\left(\frac{a}{2}\right)^{2n-1} \sum_{k=1}^{n} \binom{2n-1}{n-k} \mathbf{u}_{1,2k-1}^{(e)} \tag{1.6}$$

$$\mathbf{u}_{1,2n}^{(p)} = \left(\frac{a}{2}\right)^{2n} \sum_{k=-n}^{n} \binom{2n}{n-k} \mathbf{u}_{1,2k}^{(e)}$$

$$\mathbf{u}_{2,2n-1}^{(p)} = 2\left(\frac{a}{2}\right)^{2n-1} \sum_{k=1}^{n} \binom{2n-1}{n-k} [2\mathbf{u}_{2,2k-1}^{(e)} + i(2n-1-\varkappa)\mathbf{u}_{1,2k-1}^{(e)}]$$

$$\mathbf{u}_{2,2n}^{(p)} = \left(\frac{a}{2}\right)^{2n} \sum_{k=-n}^{n} \binom{2n}{n-k} [2\mathbf{u}_{2,2k}^{(e)} + i(2n-\varkappa)\mathbf{u}_{1,2k}^{(e)}]$$

$$(0 \leqslant \xi < \infty)$$

$$\mathbf{u}_{3,2k-1}^{(e)} = \sum_{n=k}^{\infty} \frac{2k-1}{2n-1}\left(\frac{a}{2}\right)^{2n-1} \binom{2n-1}{n-k} \mathbf{u}_{3,2n-1}^{(p)}$$

$$\mathbf{u}_{3,2k}^{(e)} = \sum_{n=k}^{\infty} \frac{k}{n}\left(\frac{a}{2}\right)^{2n} \binom{2n}{n-k} \mathbf{u}_{3,2n}^{(p)}$$

$$\mathbf{u}_{4,2k-1}^{(e)} = \frac{1}{2} \sum_{n=k}^{\infty} \frac{2k-1}{2n-1}\left(\frac{a}{2}\right)^{2n-1} \times \binom{2n-1}{n-k} [\mathbf{u}_{4,2n-1}^{(p)} - i(2n-1+\varkappa)\mathbf{u}_{3,2n-1}^{(p)}]$$

$$\mathbf{u}_{4,\,2k}^{(e)}=\frac{1}{2}\sum_{n=k}^{\infty}\frac{k}{n}\left(\frac{a}{2}\right)^{2n}\binom{2n}{n-k}[\mathbf{u}_{1,\,2n}^{(p)}-i(2n+\varkappa)\,\mathbf{u}_{3,\,2n}^{(p)}]$$
$$(\rho>a)$$

Expansions of the basis solutions of the Laplace equation in elliptic and polar coordinates were used in deriving relationships (1.6)

$$e^{-2k\xi}e^{2ik\theta}=\sum_{n=k}^{\infty}\frac{k}{n}\left(\frac{a}{2}\right)^{2n}\binom{2n}{n-k}\rho^{-2n}e^{2in\varphi}$$

$$e^{-(2k-1)\xi}e^{i(2k-1)\theta}=\sum_{n=k}^{\infty}\frac{2k-1}{2n-1}\left(\frac{a}{2}\right)^{2n-1}\binom{2n-1}{n-k}\rho^{-(2n-1)}e^{i(2n-1)\varphi}$$
$$(\rho>a)$$

$$\rho^{2n}e^{2in\varphi}=\left(\frac{a}{2}\right)^{2n}\sum_{k=-n}^{n}\binom{2n}{n-k}\operatorname{ch}2k(\xi+i\theta)$$

$$\rho^{2n-1}e^{i(2n-1)\varphi}=2\left(\frac{a}{2}\right)^{2n-1}\sum_{k=1}^{n}\binom{2n-1}{n-k}\operatorname{ch}(2k-1)(\xi+i\theta)$$
$$(\xi\geqslant 0)$$

The method of obtaining expansions of this kind and their application to the solution of scalar boundary value problems is described in /1, 2/.

In combination with the Fourier method, relationships (1.6) are especially adapted to the solution of vector boundary problems of elasticity theory for a piecewise-homogeneous medium $0\leqslant\rho\leqslant\rho_0$ with concentric circular lines of separation $\rho=\rho_s$ $(s=1,2,\ldots,m)$ comprising its homogeneous domains and the crack $\xi=0$ in the inner domain $0\leqslant\rho\leqslant\rho_m$ relationships (1.6) themselves are used to satisfy the conjugate conditions on the circle $\rho=\rho_m$ and the boundary conditions on the edges of the crack $\xi=0$. The outer contour $\rho=\rho_0$ can be missing $(\rho_0=\infty)$, corresponding to the case of a piecewise-homogeneous plane. If the principal vector of the forces applied to the edges of the crack is different from zero here, then the elementary solution (1.5) must be included in the general solution of (1.1) for the domain $\rho_1<\rho<\infty$.

The realization of such an approach results in infinite systems of linear algebraic equations of the second kind with exponentially decreasing matrix coefficients.

2. We will apply relationships (1.6) to the solution of the problem of the state of stresses of a piecewise-homogeneous plane $0\leqslant\rho<\infty$ with circular lines of separation $\rho=\rho_1$ of rigidly adherent homogeneous domains and the crack (slit) $\xi=0$ in the inner domain $0\leqslant\rho\leqslant\rho_1$.

We will confine ourselves to an examination of two loading cases: a) a normal uniformly distributed pressure of intensity σ_0 is applied to the crack edges, and b) the edges at the centre of the crack are stretched by normal concentrated forces P.

The conjugate conditions on the line $\rho=\rho_1$ and the boundary conditions on the crack edges $\xi=0$ then have the form

$$u_\rho^{(1)}=u_\rho^{(2)},\quad u_\varphi^{(1)}=u_\varphi^{(2)},\quad \tau_{\rho\varphi}^{(1)}=\tau_{\rho\varphi}^{(2)},\quad \sigma_\rho^{(1)}=\sigma_\rho^{(2)}\quad(\rho=\rho_1)\tag{2.1}$$
$$\tau_{xy}^{(1)}=0;\quad \text{a) }\sigma_y^{(1)}=-\sigma_0;$$
$$\text{b) }\sigma_y^{(1)}=-\frac{P}{a}\left[\delta\left(\frac{\pi}{2}-\theta\right)+\delta\left(\frac{3}{2}\pi-\theta\right)\right]\quad(\xi=0)$$

Here $\sigma_\rho^{(i)}$, $\tau_{\rho\varphi}^{(i)}$, $\tau_{xy}^{(1)}$, $\sigma_y^{(1)}$, $u_\rho^{(i)}$, $u_\varphi^{(i)}$ are stress tensor components and vector displacement components in polar and Cartesian coordinates, $\delta(x)$ is the Dirac delta-function and the subscripts 1 and 2 refer to the domains $D_1=(0\leqslant\rho\leqslant\rho_1)$ and $D_2=(\rho_1\leqslant\rho<\infty)$, respectively.

Taking account of the symmetry of the problem in the x, y coordinates, we represent the general solutions of (1.1) for the domains D_1 and D_2 in the form

$$\mathbf{u}^{(1)}=\sum_{k=1}^{\infty}A_k^{(1)}\operatorname{Re}\mathbf{u}_{3,\,2k-1}^{(e)}+\sum_{k=1}^{\infty}A_k^{(2)}\operatorname{Im}\mathbf{u}_{4,\,2k-1}^{(e)}+\sum_{n=2}^{\infty}B_n^{(1)}\operatorname{Re}\mathbf{u}_{1,\,2n-3}^{(p)}+\sum_{n=1}^{\infty}B_n^{(2)}\operatorname{Im}\mathbf{u}_{2,\,2n-1}^{(p)}\quad(\nu=\nu_1)$$

$$\mathbf{u}^{(2)}=\sum_{n=1}^{\infty}D_n^{(1)}\operatorname{Re}\mathbf{u}_{3,\,2n-1}^{(p)}+\sum_{n=2}^{\infty}D_n^{(2)}\operatorname{Im}\mathbf{u}_{4,\,2n-3}^{(p)}\quad(\nu=\nu_2)$$

Satisfying conditions (2.1) and using relationships (1.6) here, we obtain an infinite system of linear algebraic equations in the coefficients A_k^2 after simple reduction

$$A_k^{(2)} = \sum_{m=1}^{\infty} D_{km}(\lambda) A_m^{(2)} + F_k \quad (k = 1, 2, \ldots) \tag{2.2}$$

$$D_{11} = \frac{1}{4}(p_1 + 2p_2)\lambda^2 + 4\sum_{n=2}^{\infty} \frac{1}{2n-1}\binom{2n-1}{n-1}^2 \times$$
$$\left[\frac{1}{4}p_1 + p_3(n - \omega_{n1}\lambda^{-2})^2\right]\left(\frac{\lambda}{2}\right)^{4n-2}$$
$$D_{km} = 4\sum_{n=\max(k,m)}^{\infty} \frac{2m-1}{2n-1}\binom{2n-1}{n-k}\binom{2n-1}{n-m} \times$$
$$\left[\frac{1}{4}p_1 + p_3(n - \omega_{nk}\lambda^{-2})(n - \omega_{nm}\lambda^{-2})\right]\left(\frac{\lambda}{2}\right)^{4n-2}$$
$$p_1 = \frac{G_1\varkappa_2 - G_2\varkappa_1}{G_1\varkappa_2 + G_2}, \quad p_2 = \frac{G_1 - G_2}{G_1 + G_2(1 - 2\nu_1)}, \quad p_3 = \frac{G_1 - G_2}{G_1 + G_2\varkappa_1}$$
$$\omega_{nj} = \frac{(n-j)(n+j-1)}{n-1}, \quad \lambda = \frac{a}{\rho_1} < 1$$
$$\text{a) } F_1 = -\frac{a\sigma_0}{2G_1}, \quad F_n = 0\ (n = 2, 3, \ldots); \quad \text{b) } F_k = \frac{P}{\pi G_1}\frac{(-1)^k}{2k-1}$$

The remaining coefficients are determined from the formulas

$$A_k^{(1)} = (1 - 2\nu_1)A_k^{(2)}, \quad \rho_1 B_1^{(2)} = \frac{1}{4}p_2\lambda A_1^{(2)},$$
$$\rho_1 D_1^{(1)} = -\frac{G_1(1-\nu_1)\lambda A_1^{(2)}}{G_1 + G_2(1 - 2\nu_1)}$$
$$\rho_1^{2n-3}B_n^{(1)} = p_1\Sigma_1 - p_3(2n-1)\Sigma_2, \quad \rho_1^{2n-1}B_n^{(2)} = p_3\Sigma_2$$
$$\rho_1^{-(2n-1)}D_n^{(1)} = -\frac{G_1(1+\varkappa_1)}{G_1\varkappa_2 + G_2}(2n-3)\Sigma_1 - \frac{G_1(1+\varkappa_1)}{G_2\varkappa_1 + G_1}\Sigma_2$$
$$\rho_1^{-(2n-3)}D_n^{(2)} = \frac{G_1(1+\varkappa_1)}{G_1\varkappa_2 + G_2}\Sigma_1$$
$$\Sigma_1 = \frac{1}{4}\left(\frac{\lambda}{2}\right)^{2n-3}\sum_{k=1}^{n} A_k^{(2)}\frac{2k-1}{2n-1}\binom{2n-1}{n-k}\frac{\omega_{nk}}{2n-3}$$
$$\Sigma_2 = \left(\frac{\lambda}{2}\right)^{2n-1}\sum_{k=1}^{n} A_k^{(2)}\frac{2k-1}{2n-1}\binom{2n-1}{n-k}(n - \omega_{nk}\lambda^{-2})$$

Using the estimates

$$2^{-(4n-2)}\binom{2n-1}{n-k}\binom{2n-1}{n-m} \leqslant 2^{-(4n-2)}\binom{2n-1}{n-1}^2 = \frac{\Gamma^2(n+1/2)}{\pi\Gamma^2(n+1)} < \frac{1}{\pi}$$
$$\frac{1}{2n-1} < 1, \quad \frac{n}{2n-1} < 1, \quad \omega_{nj} \leqslant n, \quad |n - \omega_{nj}\lambda^{-2}| \leqslant n(1 + \lambda^{-2})$$

we can see that

$$|D_{11}(\lambda)| \leqslant {}^1/_4|p_1 + 2p_2|\lambda^2 + {}^1/_3 d_{22}(\lambda)\lambda^6 \tag{2.3}$$
$$|D_{km}(\lambda)| \leqslant d_{km}(\lambda)\lambda^{4\max(k,m)-2} \quad (k + m \neq 2)$$
$$d_{km} = \frac{2m-1}{\pi}\frac{(1-\lambda^2)^{-2}}{1-\lambda^4}\{|p_1|(1-\lambda^2)^2 +$$
$$4|p_3|[\max(k,m)(\lambda^{-4} - 1) + 1]\}$$

and therefore

$$\sum_{k,m=1}^{\infty} D_{km}^2(\lambda) < \infty \quad (0 < \lambda < 1) \tag{2.4}$$

It follows from inequalities (2.4) and the fact that $\{F_k\}_1^\infty$ belongs to the Hilbert space of number sequences l_2 that for almost all values of $\lambda \in (0, 1)$ the solution of the infinite system (2.2) in the space l_2 exists, is unique, and can be found by the method of reduction /3, 4/.

Moreover, it follows from estimates (2.3) that

$$S_k(\lambda) = \sum_{m=1}^{\infty} |D_{km}(\lambda)| \to 0$$

as $k \to \infty$ and $0 < \lambda < 1$, i.e., the infinite system (2.2) is quasiregular. Then taking account of the equation $S_k(0) = 0$ $(k = 1, 2, \ldots)$ it can be concluded that for any fixed set of values $G_2/G_1, \nu_1, \nu_2$ an appropriate value λ^* exists such that the infinite system (2.2) is completely regular for $0 < \lambda \leqslant \lambda^*$.

Analysis of the sum $S_k(\lambda)$ $(k = 1, 2, \ldots)$ based on finer estimates shows that in the case $G_1 \geqslant G_2$ the infinite system (2.2) is completely regular for $0 < \lambda \leqslant 1/\sqrt{2}$.

Limiting ourselves to terms up to the order λ^6 inclusive, we will have

$$\text{a)}\ A_1^{(2)} = -\frac{a\sigma_0}{2G_1}\left\{1 + \frac{1}{4}q\lambda^2 + \frac{1}{16}(q^2 - 24p_3)\lambda^4 + \frac{1}{64}[3(p_1 + 31p_3) - 48p_3q + q^3]\lambda^6\right\} + O(\lambda^8)$$

$$A_2^{(2)} = \frac{a\sigma_0}{2G_1}\left\{\frac{1}{4}p_3\lambda^4 + \frac{1}{64}[4p_3q - (p_1 + 31p_3)]\lambda^6\right\} + O(\lambda^8)$$

$$A_k^{(2)} = O(\lambda^{4k-4}) \quad (k = 3, 4, \ldots)$$

$$\text{b)}\ A_1^{(2)} = -\frac{P}{\pi G_1}\left\{1 + \frac{1}{4}q\lambda^2 + \frac{1}{16}(q^2 - 20p_3)\lambda^4 + \frac{1}{64}[2(p_1 + 31p_3) - 44p_3q + q^3]\lambda^6\right\} + O(\lambda^8)$$

$$A_2^{(2)} = \frac{P}{3\pi G_1}\left\{1 + \frac{3}{4}p_3\lambda^4 + \frac{1}{32}[6p_3q - (p_1 + 31p_3)]\lambda^6\right\} + O(\lambda^8)$$

$$A_k^{(2)} = O(\lambda^{4k-4})\ (k = 3, 4, \ldots);\ q = p_1 + 2p_2 + 3p_3$$

The stress intensity coefficients calculated on the basis of the asymptotic solutions a) and b) have the form

$$\text{a)}\ k = \sigma_0\sqrt{a}\,\{1 + 1/4q\lambda^2 + 1/16\,(q^2 - 36p_3)\lambda^4 + 1/64\,[q^3 - 60p_3q + 6(p_1 + 31p_3)]\lambda^6\} + O(\lambda^8) \tag{2.5}$$

$$\text{b)}\ k = \frac{P}{\pi\sqrt{a}}\left\{1 + \frac{1}{2}q\lambda^2 + \frac{1}{8}(q^2 - 32p_3)\lambda^4 + \frac{1}{32}[q^3 - 56p_3q + 4(p_1 + 31p_3)]\lambda^6\right\} + O(\lambda^8)$$

In the special case when $G_2 = 0$ the expressions in the braces in (2.5) are: a) $1 + 3/2\lambda^2 + 3/4\lambda^6$, b) $1 + 3\lambda^2 + 1/2\lambda^4 + 1/4\lambda^6$ i.e., we obtain asymptotic formulas that agree with the expressions for the stress intensity coefficients in the problem of the equilibrium of a homogeneous circular disc with a symmetrical diametral crack /5, 6/.

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Translated by M.D.F.